Interferometric Measurements Using Redundant Phase Centers of Synthetic Aperture Sonars

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***Abstract-* Interferometric sonars with multiple horizontal rows of elements have been used routinely to produce swath bathymetry. However, interferometric sonars are larger more complex, and consume more power than arrays with a single row of elements. Synthetic aperture sonar (SAS) systems often require the use of redundant phase centers (RPC), where the aft sonar element positions overlap in space with the forward element positions of the previous ping. Considering that a vehicle carrying a SAS array would likely have non-zero pitch, the use of RPC provides sonar data from receivers at the same along-track position with some vertical displacement. This data is similar to that of interferometric systems with the exception that the distance between receiver pairs can vary with vehicle motion and the received signals are not collected concurrently.**

This paper evaluates the possibility that an interferometric capability could be achieved using RPC data collected from a SAS system consisting of a single horizontal row of elements. An error analysis was conducted to determine the effect of errors in relative receiver position on swath bathymetry.

Results show that errors in receiver vertical displacement result in similar percent errors in elevation. Therefore, errors in swath bathymetry can be reduced by designing the array to increase vertical displacement between RPC pairs. Results also show that increasing vertical displacement between RPC pairs can also reduce the impact of data phase measurement errors on swath bathymetry. Swath bathymetry measurements are very sensitive to errors in across-track displacements, but the predictable nature and scale of the error may indicate that accurate across-track displacements could be calculated from phase measurements. Swath bathymetry images produced from data acquired by an existing SAS consisting of a single horizontal row of elements are shown and illustrate viability of the technique depending on the required resolution of the system.

I. Introduction

Interferometric sonars with multiple horizontal rows of elements have an advantage over single-row systems in that changes in the path length to a target caused by vertical displacement of receiver position can be measured [1], [2], [3]. This measured change in path length can be used to calculate the elevation of targets in a sonar return and is often used to construct swath bathymetry. However, interferometric sonars with multiple rows of elements are larger, more complex, and consume more power than arrays with a single horizontal row of elements. This makes interferometric systems difficult to fit on small-format vehicles.

Synthetic aperture sonar (SAS) systems require that the vehicle motion must be within acceptable limits and the vehicle position relative to the image surface must be accurately measured to provide images at the maximum achievable resolution. These high-fidelity motion estimates often require the use of redundant phase centers (RPC), where the aft sonar element positions overlap in space with the forward element positions of the previous ping. Considering that a vehicle carrying a SAS array would likely have non-zero pitch, the use of redundant phase centers results in collection of sonar data from multiple receivers at the same along-track position with some vertical displacement. The data would be similar to that from interferometric systems with the exception that the distance between receiver pairs can vary with vehicle motion and the received signals are not collected concurrently. However, if the relative positions of the RPC elements are known, RPC data could be used to emulate the response of an interferometric system.

Practical application of this technique is complicated by uncertainty in RPC element relative displacement in two dimensions (vertical and across-track displacement). Determination whether this technique is possible with present navigation capabilities is required to justify development of a new system or modification of an existing system to add a swath bathymetry capability. This paper evaluates the possibility that an interferometric capability could be achieved with an array consisting of a single horizontal row of elements if element positions overlap between pings and the array has some vertical displacement between overlapping element pairs. An error analysis was conducted to determine the effect of errors in relative receiver position on swath bathymetry. Estimates of required navigational accuracy are discussed, and swath bathymetry images produced from data acquired by an existing SAS system consisting of a single horizontal row of elements are shown.

II. Receiver Geometry

The technique described here to use RPC data to estimate swath bathymetry of the seafloor is similar to traditional interferometric techniques in that the elevation of a target is estimated by the difference in path length for two independent sonar receivers. The fundamental difference between this technique and traditional interferometric systems is that data from

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14. ABSTRACT

Interferometric sonars with multiple horizontal rows of elements have been used routinely to produce swath bathymetry. However, interferometric sonars are larger more complex, and consume more power than arrays with a single row of elements. Synthetic aperture sonar (SAS) systems often require the use of redundant phase centers (RPC), where the aft sonar element positions overlap in space with the forward element positions of the previous ping. Considering that a vehicle carrying a SAS array would likely have non-zero pitch, the use of RPC provides sonar data from receivers at the same along-track position with some vertical displacement. This data is similar to that of interferometric systems with the exception that the distance between receiver pairs can vary with vehicle motion and the received signals are not collected concurrently. This paper evaluates the possibility that an interferometric capability could be achieved using RPC data collected from a SAS system consisting of a single horizontal row of elements. An error analysis was conducted to determine the effect of errors in relative receiver position on swath bathymetry. Results show that errors in receiver vertical displacement result in similar percent errors in elevation. Therefore, errors in swath bathymetry can be reduced by designing the array to increase vertical displacement between RPC pairs. Results also show that increasing vertical displacement between RPC pairs can also reduce the impact of data phase measurement errors on swath bathymetry. Swath bathymetry measurements are very sensitive to errors in across-track displacements, but the predictable nature and scale of the error may indicate that accurate across-track displacements could be calculated from phase measurements. Swath bathymetry images produced from data acquired by an existing SAS consisting of a single horizontal row of elements are shown and illustrate viability of the technique depending on the required resolution of the system.

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the two receivers is separated in time as well as the spatial separation, and that spatial separation between the elements will vary between measurement pairs.

The system geometry is shown in Figure 1, where R1 and R2 represent two receivers who's relative positions are displaced by Δz in elevation and Δy in the ground-plane range direction. The path length from receiver positions R1 and R2 to a target T are described in (1) and (2), which also define the change in path length for R2 (Δs) resulting from y and z displacements. Δs can be estimated from the difference in phase of data received by the two receivers [1], [2].

$$s = (y^2 + z^2)^{0.5} \tag{1}$$
$$s + \Delta s = ((z + \Delta z)^2 + (y + \Delta y)^2)^{0.5} \tag{2}$$

Equations (1) and (2) can be solved for z yielding a quadratic equation that can be solved by (3), where A, B, and C are defined by (4)–(6).

$$z = (-B \pm (B^2 - 4AC)^{0.5})/2A \tag{3}$$
$$A = 4(\Delta y^2 + \Delta z) \tag{4}$$
$$B = 4(\Delta z^3 + \Delta y^2\Delta z - \Delta s^2\Delta z - 2s\Delta s\,\Delta z) \tag{5}$$
$$C = 4s(s\Delta s^2 + s\Delta y^2 + \Delta s^3 - \Delta s\Delta z^2 - \Delta s\Delta y^2) - 2(\Delta s^2\Delta y^2 + \Delta s^2\Delta z^2 - \Delta y^2\Delta z^2) + \Delta s^4 + \Delta y^4 + \Delta z^4 \tag{6}$$

This solution for z can be simplified by ignoring terms including third order or higher displacements (i.e. combinations of Δs, Δy, or Δz that are third or fourth order). These third or fourth order displacement terms are on the order of a wavelength cubed (λ^3) and contribute very little to the estimate of z. The resulting simplified solution for z is shown in (7).

$$z = (s\Delta s\Delta z \pm s\Delta y(\Delta y^2 + \Delta z^2 - \Delta s^2)^{0.5})/(\Delta y^2 + \Delta z^2) \tag{7}$$

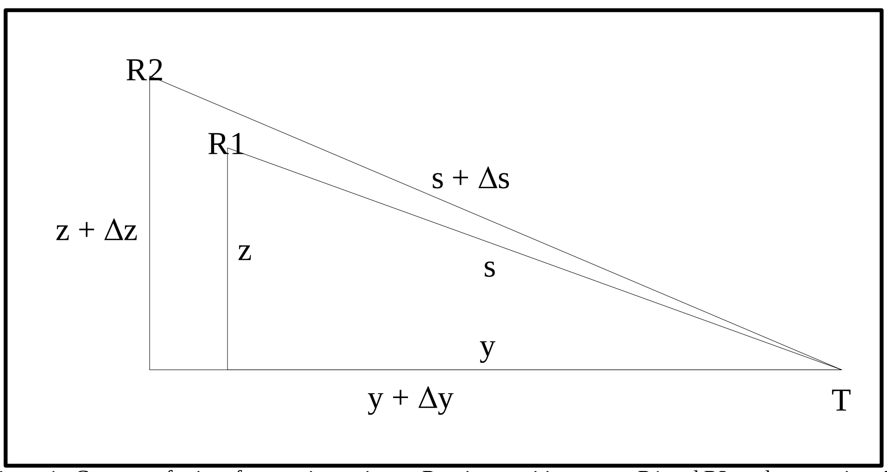

Figure 1: Geometry for interferometric receivers. Receiver positions are at R1 and R2, and a target is at T. s corresponds to slant range, z corresponds to elevation, and y corresponds to ground-plane range.

III. Error Analysis

Error analysis addressed in this paper will focus on aspects of interferometry unique to the RPC method described above. Primarily, error in the relative position of the two receivers is discussed. Other factors contributing to errors in interferometric bathymetry derived from static relative receiver positions have been thoroughly discussed by Lurton [4], and Bellettini and Pinto [5].

Errors associated with receiver relative positions were evaluated through several techniques. Simplified geometric solutions which consider displacement in only one direction (i.e. only Δy or Δz displacement) were evaluated through simplification of (7). Errors associated with displacement in two dimensions are evaluated by introducing errors in Δy or Δz measurements and calculating the associated error in z from (3). Nominal values for a SAS system were assumed for this calculation, including that the bottom is completely flat with R1 at 5m altitude and R2 displaced by 0.015m in z and y (Δy and Δz equal to 1λ at 100kHz).

Error in z-displacement

The resulting error in z caused by an error in z displacement for a case with no y displacement can be evaluated by simplifying (7). Assuming $\Delta y = 0$, (7) reduces to (8). Introducing an error of $e_{\Delta z}$ into the equation results in an error in z (e_z). Solving (9) for e_z and dividing by (8) results in an expression that compares percent errors in Δz to the resulting percent error in z. This relationship shows that errors in Δz correspond in near parity to errors in z. For example, a 10%

error in Δz would result in an error in z of 9.1%. This relationship is independent of range.

$$z = s\,\Delta s/\Delta z \tag{8}$$
$$z + e_z = s\,\Delta s/(\Delta z + e_{\Delta z}) \tag{9}$$
$$e_z/z = -e_{\Delta z}/(\Delta z + e_{\Delta z}) \tag{10}$$

The simple relationship described by (10) becomes more complicated with the introduction of y displacement. Assuming z and y displacements of 0.015m, an R1 altitude of 5m, and z displacement errors of 1 and 5%, errors in z were calculated for slant ranges of 10–100m (Figure 2). The results show that the percent error in z approaches that of the case where $\Delta y = 0$ at long range, but resulting errors in z are much higher at near range.

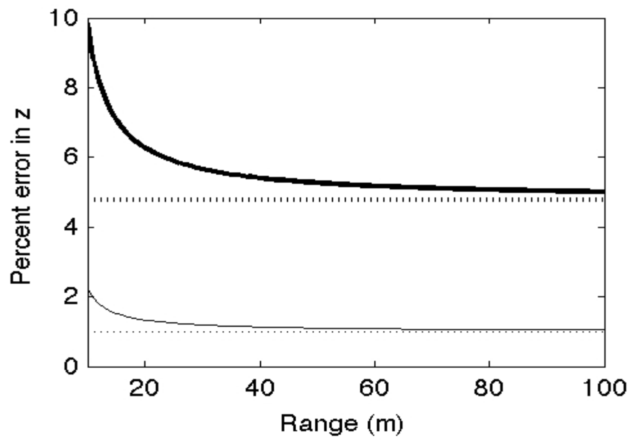

Figure 2: Percent error in z due to a 1% error (thin line) and a 5% error (bold line) in z displacement assuming an R1 altitude of 5m and R2 displaced from R1 by 0.015m in y and z. Percent error in z for 1% and 5% error in z displacement with no y displacement is shown for reference (dotted).

Error in y-displacement

The resulting error in z caused by an error in y displacement for a case with no z displacement was evaluated in a similar process as that completed for errors associated with z displacements. Equation (7) was simplified assuming that $\Delta z = 0$ (11). An error ($e_{\Delta y}$) was introduced resulting in an error in z (e_z) (12). Solving (12) for e_z demonstrates that e_z increases in magnitude continuously with increasing slant range (13).

$$z = (\pm s\Delta y(\Delta y^2 - \Delta s^2)^{0.5})/\Delta y^2 \tag{11}$$
$$z + e_z = (\pm s(\Delta y + e_{\Delta y})((\Delta y + e_{\Delta y})^2 - \Delta s^2)^{0.5})/(\Delta y + e_{\Delta y})^2 \tag{12}$$
$$e_z = -z \pm s(1 - (\Delta s/(\Delta y + e_{\Delta y}))^2)^{0.5} \tag{13}$$

Analysis of the case where z and y displacement are considered shows results consistent with the relationship predicted by (11) with the exception that the rate of increase is lower for the cases with $\Delta z = 0.015$m (Figure 3). This reduction of the impact of errors in Δy is due to the relative importance of the term $s\Delta s\Delta z$ in (7), which increases in magnitude as Δz increases.

Error in phase measurement

Errors in bathymetry associated with errors in phase measurement due to both hardware limitations and physical phenomena have been thoroughly discussed in literature concerning interferometric sonars [4], [5]. However, standard interferometric sonars have static relative receiver positions and the techniques proposed in this paper involve phase centers that vary in displacement due to vehicle motion. Therefore, the combined effect of changes in Δz and uncertainty in phase was evaluated.

A Monte Carlo simulation was performed to evaluate the effect of a randomly generated phase error on the resulting z. The phase error was randomly generated from a normal distribution with a standard deviation of π/64–π/4 radians (π/4 radians corresponding approximately to a 8dB signal SNR [4]). The error in z was determined by calculating the expected phase predicted by (3), adding the phase error to the predicted Δs, and calculating z from (3). The process was repeated 10,000 times, and the standard deviation of the resulting errors in z are shown in Figure 4. The simulation was repeated for values of Δz ranging from 1λ (0.015m) to 6λ. The results shown in Figure 4 indicate that higher uncertainty in phase leads to large errors in z for small receiver vertical separations, but errors become less pronounced as receiver vertical separation increases. The effect of receiver separation is less apparent when phase uncertainty is low.

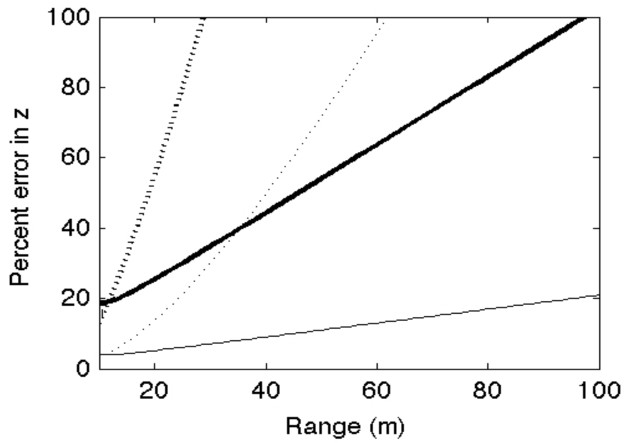

Figure 3: Percent error in z due to a 1% error (thin line) and a 5% error (bold line) in y displacement assuming an R1 altitude of 5m and R2 displaced from R1 by 0.015m in y and z. Percent error in z for 1% and 5% error in y displacement with no z displacement is shown for reference (dotted).

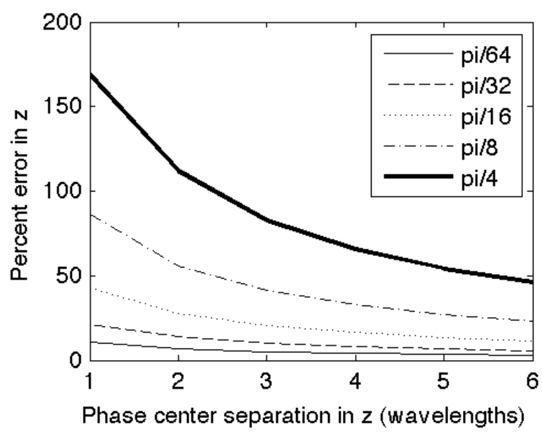

Figure 4: Percent error in z due to errors in phase measurements assuming an R1 altitude of 5m and R2 displaced from R1 by 0.015m in y and varying in z (1-6 wavelengths). Results were generated using a Monte Carlo simulation. Values shown for phase errors represent the standard deviation of a normally distributed set of random errors, and the resulting percent error in z is the standard deviation of the resulting simulated z-value errors.

IV. Examples of Swath Bathymetry

The hypothesis that swath bathymetry could be constructed from RPC element pair data was tested by producing images and swath bathymetry for two data sets acquired by an existing SAS system with a single horizontal row of receivers. The two data sets were selected based on apparent topography evident in synthetic aperture imagery (Figure 5 A and B) and vehicle navigation characteristics. The topography for the first data set appears flat and relatively smooth (Figure 5A), while topography for the second set has obvious structures on the bottom (Figure 5B). However, these data sets were not collected for the purpose of interferometry and the seafloor topography could not be independently verified.

Navigation characteristics used to select the data sets included identifying data with large vertical displacement and small across-track displacement of RPC element pairs. The data set with an apparent flat bottom had a mean relative vertical displacement of RPC element pairs of 1.4λ and a mean relative across-track displacement of RPC element pairs of –0.1λ, where λ corresponds to the wavelength at the sonar center frequency. The data set with apparent structures on the bottom had a mean relative vertical displacement of RPC element pairs of 1.7λ and a mean relative across-track displacement of RPC element pairs of –0.6λ. The data sets were also collected at different vehicle altitudes: the first at 5.3m, and the second at 3.6m.

SAS images were generated to reference bottom features likely to be present in swath bathymetry (Figure 5 A and B). In each image, the vehicle path can be located as the line vertically through the center of the image and is obvious due to the presence of the nadir. Figure 5A shows a flat sandy bottom with some small ripples. Figure 5B shows obvious structures in the image: clumps of debris in the bottom left corner, clumps of debris diagonally across the bottom right corner, a clump of debris near the top of the image that is most prominent on the right, but is present to some degree across the image including a small “bump” in the nadir.

The swath bathymetry plots created from the data sets are shown in Figures 6 and 7 and correspond to SAS imagery shown in figures 5A and 5B respectively. These examples of swath bathymetry were processed as a real aperture consisting of only RPC element pair data using (3) and were smoothed by averaging to $1m^2$ pixels. Figure 6 appears relatively smooth with an average bottom return around 5m. Figure 7 shows some variation in bottom structure with an average bottom return around 3.5m. Figure 7 also shows disturbances in the bottom contour that coincide with structures apparent in Figure 5B including the debris in the bottom corners and a smaller mound present across the top of the image.

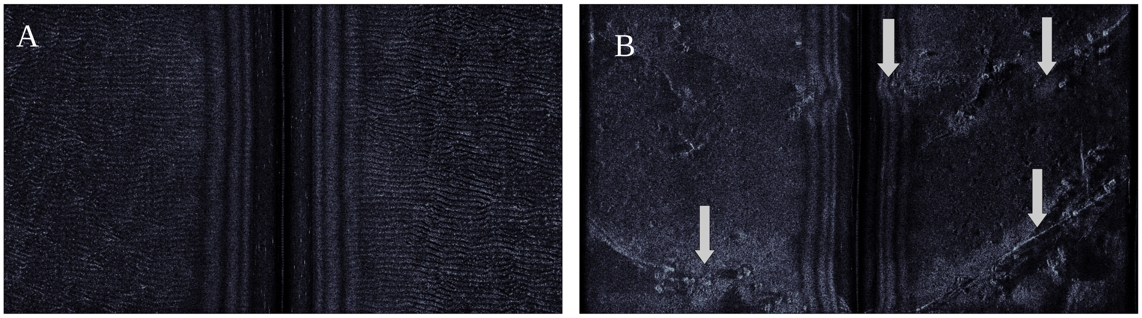

Figure 5: SAS imagery from two example data sets. One with a relatively flat bottom (A) and one with debris (B).

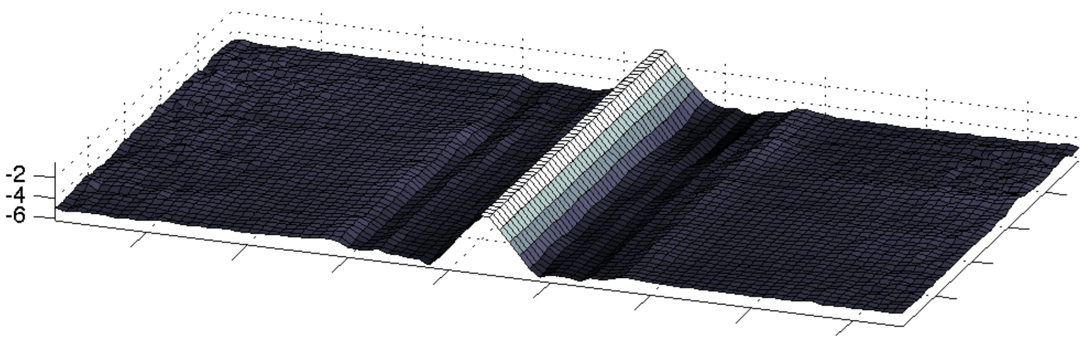

Figure 6: Swath bathymetry corresponding to imagery presented in Figure 5A.

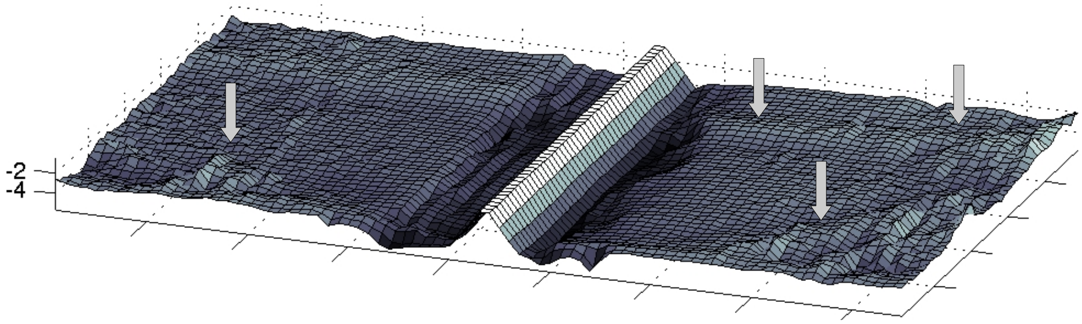

Figure 7: Swath bathymetry corresponding to imagery presented in Figure 5B. Grey arrows are included to provide to reference structures in both images.

V. Discussion

While interferometry from RPC element pair data acquired by a sonar with a single horizontal row of elements is possible if the relative receiver positions are known exactly, the accuracy of the relative receiver positions is critical to providing accurate swath bathymetry. The results from this analysis shows that errors in z displacement result in similar percent errors in calculated elevations, while small y-displacement errors can dramatically alter results. However, design of a sonar array specifically to provide bathymetry in addition to imaging capabilities could mitigate the effects of erroneous RPC element pair relative position measurements and phase uncertainty.

The effect of errors in z displacement can be mitigated through altering the array element positions to increase the vertical offset for RPC element paris, and the phase error analysis demonstrated that increasing z displacement will also reduce errors associated with phase measurement error. If the system is designed with multiple RPC element pairs, this mitigation technique could be enhanced further by varying the vertical displacement between RPC element pairs. For example, one RPC element pair could be designed to produce a 1λ vertical separation, while a second RPC element pair could be designed to produce a 2λ vertical separation. Data from multiple RPC element pairs with differing z displacement measurements could be used with (1) and (2) to explicitly solve for Δz rather than relying on deriving this value from vehicle navigation parameters. Moreover, data from multiple RPC element pairs with differing z displacements could also be used to resolve ambiguity associated with phase unwrapping, similar to techniques applied to interferometric systems with more than two rows of receivers.

Although increasing the vertical displacement of RPC receivers would moderate swath bathymetry errors associated with erroneous across-track displacement measurements (Figure 3), it is unlikely that the effect of errors in across-track displacement can be satisfactorily resolved through improving navigational accuracy or array modification. Therefore, techniques to measure the across-track displacement from the RPC data should be developed and refined to provide the necessary accuracy. This could be accomplished with (2) by substituting in an assumed value for z and solving for Δy (given Δs and Δz from sonar and navigation data).

VI. Conclusions

While the procedure to produce swath bathymetry from RPC element pair data is technically challenging, it is possible to include swath bathymetry as a capability of small-format SAS arrays without increasing the number of elements or array complexity. However, development of techniques to accurately measure vertical and across-track displacement of RPC element pairs is critical to successful generation of swath bathymetry. Receiver array design can potentially provide increased accuracy in vertical and across-track displacement of RPC element pairs and reduce the impact of residual displacement errors. SAS array design options should be explored to determine the design parameters necessary for the most accurate vertical and across-track displacement estimates from RPC element pair data, and techniques to estimate RPC element displacements from sonar data should be developed. Once developed, these improved RPC displacement measurements could be used to improve vehicle motion estimates and the SAS imagery as well as the swath bathymetry.

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References

[1] M. Pinto, R.D. Hollett, A. Bellettini, and S. Chapman, "Bathymetric imaging with wideband interferometric synthetic aperture sonar," *Submitted to IEEE J. Oceanic Eng*., 2001.

[2] T.O. Saebo, and R.E. Hansen, "Bathymetric Imaging Using Synthetic Aperture Sonar," *Proceedings of the Seventh European Conference on Underwater Acoustics, ECUA 2004* Delft, The Netherlands, pp.1157-1162, 5-8 July, 2004.

[3] T. O. Saebo, R.E. Hansen, and H.J. Callow, "Height Estimation on Wideband Synthetic Aperture Sonar: Experimental Results from InSAS-2000," *OCEANS 2005 Proceedings of MTS/IEEE*, vol. 3, pp. 2782-2788, 2005.

[4] X. Lurton, "Swath Bathymetry Using Phase Difference: Theoretical Analysis of Acoustical Measurement Precision," *IEEE Journal Of Oceanic Engineering*, vol. 25(3), pp. 351-363, July 2000.

[5] A. Bellettini, and M.A. Pinto, "Theoretical Accuracy of Synthetic Aperture Sonar micronavigation Using a Displaced Phase-Center Antenna," *IEEE Journal of Oceanic Engineering*, vol.27 (4), 780-789, October 2002.